Information and Communication in Organizations

By Inga Deimen and Dezső Szalay*

Information is vital for good decision-making in organizations. However, decision-makers often have to rely on others to provide them with information. Therefore, information has to travel through the organization to the decision-maker.

Our model highlights two potential reasons why the quality of information can diminish on its way to the decision-maker. Firstly, individuals who see the information draw inferences from it, that is they process the information statistically and reduce the informational content to what is relevant to themselves only. This is a completely non-strategic procedure in which the individual forms a posterior expectation from the observed facts. Since the posterior expectation, henceforth the inference, is all that matters to the individual, not more than this statistical piece of information gets passed on to the decision-maker. Secondly, there may be strategic reasons to withhold information from decision-makers because there may be disagreement about how to use the information in decision-making.

In this paper, we identify a benchmark in which the second component is absent in equilibrium. In particular, we take a constrained information design perspective (see Kamenica and Gentzkow (2011) for an analysis of the completely unconstrained persuasion problem), where a designer controls the noise in the observations of the individual who gets to see the information first – the sender. The designer's objective is to maximize the sum of both individuals' payoffs, taking into account that the sender gets to see the information first, before he passes it on to the second individual – the receiver – who takes the decision that affects both individuals' payoffs. Noise is the only instrument available to the designer to influence decision-making.

It turns out that controlling the noise in the sender's observations is a surprisingly powerful tool. The choice of information is quite similar to a risk-sharing problem within the organization. When both individuals face equal uncertainty ex ante and are sufficiently averse to errors in the decision-making process, then the designer's optimal choice of information structure equalizes the residual uncertainty that the sender and the receiver face. Moreover, there is no disagreement on the use of the statistically processed information between the sender and the receiver. Hence, given the optimal information, there is no way to improve the decision-making procedure: a reallocation of decision-making rights from the receiver to the sender would not affect the optimal decision-rule. In other words, the optimal provision of information and the allocation of authority are substitutes in our model.

The idea of information processing in organizations has already been discussed by Cyert and March (1963), preceding the seminal paper by Crawford and Sobel (1982) on the strategic transmission of information. We emphasize that the informational loss in the present model is of a different nature than in the Crawford and Sobel (1982) model. The only loss is due to a statistical processing of the information, because our sender's conditionally optimal action just depends on an aggregate of the observed signals. In companion work (Deimen and Szalay (2019)) we study the information design problem when the sender has reasons to strategically withhold informa-

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The present paper is one in two spin-offs of Deimen and Szalay (2016). Many thanks to Marina Halac, Navin Kartik, and Vasiliki Skreta.

tion as in Crawford and Sobel (1982). In Deimen and Szalay (forthcoming), we consider a sender-receiver game and identify situations in which the sender never has incentives to acquire information that would misalign his interests from the receiver's at the communication stage.

I. Model

We consider a strategic interaction between three parties in fixed roles, a sender (he), a receiver (she), and a designer. For example, the sender and the receiver could correspond to divisions of a firm while the designer could correspond to the headquarters of the firm. A decision $x \in \mathbb{R}$ needs to be taken that affects the payoffs of all three parties. The sender has preferences described by

$$u^s(x,\eta) = -\ell(x-\eta);$$

the receiver has preferences

$$u^r(x,\omega) = -\ell(x-\omega).$$

The loss function $\ell(q)$ is symmetric around its minimizer, $q = 0$, twice differentiable, and at least as convex as the quadratic function. More precisely, we assume that the Arrow-Pratt measure of the relative curvature of the loss function satisfies $\frac{q\ell''(q)}{\ell'(q)} \geq 1$ for all $q \neq 0$. In addition, ℓ rises sufficiently slowly to make expected utility well-defined.[1] The ideal policies of the players are given by $x^r(\omega) = \omega$ and $x^s(\eta) = \eta$, for the receiver and the sender respectively. The realizations of ω and η are unknown at the outset. The designer is interested in maximizing the joint surplus

$$u^d(x,\eta,\omega) = -\ell(x-\eta) - \ell(x-\omega).$$

The decision process is organized as follows. The sender gets to observe noisy signals

$$s_\omega = \omega + \varepsilon_\omega \quad \text{and} \quad s_\eta = \eta + \varepsilon_\eta,$$

where ε_ω and ε_η are noise terms that are independent of each other and of ω and η. The receiver is in charge of making the decision x. The designer can influence the interaction by choosing the information structure, that is the amount of noise in the sender's signals, $\sigma^2_{\varepsilon_\omega}$ and $\sigma^2_{\varepsilon_\eta}$ (the variances of the noise terms ε_ω and ε_η). Since it turns out to be without loss of generality, we focus on the case $\sigma^2_{\varepsilon_\omega} = 0$. The choice of the information structure is publicly observable. However, the realizations of signals s_ω and s_η are privately observed by the sender. The sender communicates with the receiver, who finally chooses x. There is no cost of sending messages and the receiver is unable to commit to the action x as a function of the information she receives, so communication is modeled as cheap talk in the sense of Crawford and Sobel (1982).

We assume that $Y = (\omega, \eta, \varepsilon_\omega, \varepsilon_\eta)$ follows a joint Normal distribution. Moreover, we assume that all the differences in preferences are unsystematic and random. That is, players agree ex ante but may disagree when given new information. Formally, we have $\mathbb{E}[Y] = \underline{0}$. While the noise variances are the designer's choice, the variances of ω and η are exogenously given and denoted by σ^2. To make the game interesting, we assume that the sender's and the receiver's ideal actions are positively correlated with coefficient $\rho \in (0,1)$ and $Cov(\omega,\eta) = \rho\sigma^2$.

II. The sender's inferences

Suppose that the designer has chosen an information structure. What action would the sender ideally want to induce and, more importantly, what part of the observed information is the sender willing to share with the receiver? The sender's preferred choice given the signals s_ω and s_η is his posterior mean, which is a linear function of the signals:

$$x^s(s_\omega, s_\eta) = \mathbb{E}[\eta | s_\omega, s_\eta] = \alpha^s s_\omega + \beta^s s_\eta,$$

where α^s and β^s are weights that can be calculated from the underlying parameters and that are independent of s_ω and s_η.[2] Note that, if the receiver obtained

[1] Examples include $\ell(q) = q^{2n}$ for $n \in \mathbb{N}$.

[2] Details can be found in an online appendix.

the signals directly, she would generally choose a different action than the sender, $x^r(s_\omega, s_\eta) = \mathbb{E}[\omega | s_\omega, s_\eta]$.

Once the sender has updated the information he communicates with the receiver, that is he sends a message to the receiver. Naturally, the informativeness of the messages received depends on the level of conflict between the players. In our setup, the conflict of interest arises endogenously as a function of the information structure that is chosen. Moreover, given that there is an underlying disagreement between the players, the sender is only partially willing to share his observed information with the receiver. That is, instead of fully revealing all the signals, he is at most willing to share his posterior optimum given the information. In particular, as is formally shown in Deimen and Szalay (forthcoming) any communication equilibrium is essentially equivalent to an equilibrium where the sender only communicates a message about

$$\theta \equiv \mathbb{E}[\eta | s_\omega, s_\eta].$$

Suppose for now that the sender would communicate θ non-strategically to the receiver. What does the receiver learn and what action would she want to take? The receiver's conditionally optimal action is the posterior expectation given the information θ,[3] that is

$$x^r(\theta) = \mathbb{E}[\omega | \theta] = \frac{Cov(\omega, \theta)}{Var(\theta)} \cdot \theta.$$

The conditional expectation corresponds to the linear regression, where the slope

$$c \equiv \frac{Cov(\omega, \theta)}{Var(\theta)} \equiv \frac{C}{V}$$

measures the importance of θ relative to the receiver's ideal action ω.

LEMMA 1: *For $\sigma^2_{\varepsilon_\omega} = 0$ we have $C = \rho\sigma^2$. Moreover, V is a decreasing function of $\sigma^2_{\varepsilon_\eta}$, that takes maximal value $\overline{V} = \sigma^2$ for $\sigma^2_{\varepsilon_\eta} = 0$, and minimal value $\underline{V} = \rho^2\sigma^2$ for $\sigma^2_{\varepsilon_\eta} \to \infty$.*

[3]Since the Normal distribution is closed under linear combinations, (ω, η, θ) are also jointly Normal distributed.

By construction of the conditional expectation θ, we have $Cov(\eta, \theta) = V$ for all information structures. Therefore, information that is more useful to the sender results in a higher variance V. Intuitively, the least useful signal to the sender is when he just learns about the receiver's ideal action ω, the most useful signal is the one that reveals the state η perfectly. The bold horizontal line in the right panel of Figure 1 depicts the feasible set described in Lemma 1.

On top of the intrinsic usefulness, the noise in the sender's signal creates or resolves conflicts between the sender and the receiver. In particular, if the sender learns his ideal action, η, then the receiver would like to react to changes in θ with a propensity $c = \rho < 1$. In contrast, if the sender just learns the receiver's ideal action, ω, the receiver would like to overreact by $c = \frac{1}{\rho}$. We illustrate the conflict between the sender and the receiver for $c < 1$ in the left panel of in Figure 1.

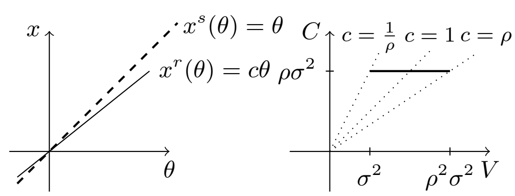

FIGURE 1.: CONFLICTS AND FEASIBILITY

III. Optimal Information

We now address the designer's problem of choosing an optimal information structure. What information should the sender get to observe? Since the sender's posterior expectation θ aggregates the sender's information into a single statistic, the designer faces a trade-off: the sender's information can be relatively more useful to the sender or the receiver, but not to both. We approach the designer's problem in two steps. We first analyze the problem purely from this statistical point of view, that is we assume that the sender's posterior expectation can be publicly observed by the receiver. In a second step, we add strategic information transmission to the picture.

A. Public Inferences

If the receiver observes the sender's inference θ, then she follows the policy $x^r(\theta) = c \cdot \theta$, resulting in a loss of $\ell(c\theta - \omega)$ for the receiver and a loss of $\ell(c\theta - \eta)$ for the sender. We note that the arguments of the loss functions, $z \equiv c\theta - \omega$ and $t \equiv c\theta - \eta$, are linear combinations of Normal variates, hence Normal as well. Let $\tilde{z} \equiv \frac{c\theta-\omega}{\sqrt{Var(c\theta-\omega)}}$ and $\tilde{t} \equiv \frac{c\theta-\eta}{\sqrt{Var(c\theta-\eta)}}$ denote the standardized arguments that follow a standard Normal with density $\phi(\cdot)$. We can write the designer's problem as

$$\max_{V\in[\underline{V},\overline{V}]} -\int \ell(z)\,\phi(\tilde{z})\,d\tilde{z} - \int \ell(t)\,\phi(\tilde{t})\,d\tilde{t}.$$

Both expected losses depend negatively on a residual variance that measures the residual uncertainty after using θ optimally from the receiver's perspective. Naturally, the residual uncertainty for the receiver is

$$(1) \qquad Var(z) = \sigma^2 - c^2V = Var(\omega|\,\theta),$$

where the second equality holds because θ is used optimally from the receiver's perspective. In contrast, θ is in general not used optimally from the sender's perspective. The residual uncertainty that the sender faces when θ is used according to the policy $x^r(\theta)$ is

$$(2) \qquad Var(t) = \sigma^2 - \left(2C - c^2V\right),$$

which differs from $Var(\eta|\,\theta) = \sigma^2 - V$ unless the optimal choices θ and $c\theta$ are identically equal to each other, $c = \frac{C}{V} = 1$.

We can now characterize the optimal information structure:

PROPOSITION 1: *If the loss function satisfies* $\frac{q\ell''(q)}{\ell'(q)} > 1$ *for all* $q \neq 0$, *then the designer's problem of choosing an optimal information structure has a unique solution, which is given by* $V^* = \rho\sigma^2$. *If the loss function satisfies* $\frac{q\ell''(q)}{\ell'(q)} = 1$ *for all* $q \neq 0$ *(corresponding to the quadratic case), then any information structure with* $\sigma^2_{\varepsilon_\omega} = 0$ *is optimal.*

The designer's problem resembles a risk sharing problem. Both players dislike higher residual uncertainty and an increase of V increases the residual uncertainty the receiver faces, equation (1), and decreases the residual uncertainty the sender faces, equation (2). For a sufficiently convex loss function, the problem has a unique maximum at the point where the residual uncertainty for both players is equalized, because the marginal loss of increasing the residual uncertainty for one player outweighs the marginal benefits for the other player. For the quadratic loss function, the designer's payoff becomes linear in the residual variances, which implies that the receiver's loss from increasing V just offsets the sender's gain and thus the sum of their payoffs becomes independent of V.

B. Private Inferences

We now consider the case of main interest where the sender has private information about θ and thus is free to make up any statement he likes. As is standard in the literature, we assume that the sender and the receiver are able to coordinate on the ex ante Pareto optimal equilibrium in the communication game. Let $m : \mathbb{R}^2_+ \times \mathbb{R}^2 \to \mathbb{M}$ denote the sender's message strategy, where $\mathbb{M}$ is sufficiently rich. Let the receiver's strategy be $x : \mathbb{R}^2_+ \times \mathbb{M} \to \mathbb{R}$. The designer's optimal choice of the information structure eliminates conflicts in a certain, well defined sense:

PROPOSITION 2: *The unique optimal information structure chosen by the designer satisfies* $V^* = \rho\sigma^2$. *The Pareto best equilibrium of the ensuing continuation game involves fully revealing communication about* θ, $m^*(\theta) = \theta$ *for all* θ, *and the receiver takes the sender's advice at face value,* $x^*(m) = m$ *for all* m.

The proof of the proposition is a straightforward combination of our preceding results and a verification that the described strategies, together with appropriate receiver beliefs, constitute an equilibrium of the communication game. Since the designer cannot improve upon its payoff compared to the case where θ is public information, the situation corresponds to an optimum if this payoff is reached. Suppose the receiver believes that the sender plays the message strategy $m(\theta) = \theta$ for all θ. Then, his best reply is the action strategy $x^*(m) = c^* \cdot m = m$ for all m. The sender, who anticipates this policy, induces his ideal policy by being fully revealing about θ, so the construction is indeed an equilibrium. Note that in this equilibrium the strategies of both players are linear functions.

COROLLARY 1: *Under the unique optimal information structure, all parties' payoffs are the same as if the sender were given the right to choose the action x directly.*

Since $x^*(m^*(\theta)) = \theta$ for all θ, the sender's optimal policy is implemented for all θ. Consequently, whether the sender communicates with the receiver or whether the sender is given the right to choose the policy, the payoffs of all parties involved are exactly the same.[4] Note that this would generally not be the case for exogenously given information, see for example Alonso, Dessein and Matouschek (2008). The intuition is that, for equal marginals, an information structure that equalizes residual uncertainty automatically eliminates any bias in the use of information. Note that there remains a conflict between sender and receiver with respect to using the underlying signals, s_ω and s_η. However, under the optimal information structure, the sender's recommendation θ and the difference $\omega - \theta$ become uncorrelated. Put differently, the optimal information structure *orthogonalizes* the conflict between the players and the recommendation and hence removes any impediments to communication.

Even if the designer or the receiver had the right to constrain the sender's discretion under delegation, they would not want to make use of this right. The sender's optimal choice is necessarily a function of his inference θ only, and the sender uses this inference in the receiver's best interest. Hence, communication is in fact unsurpassed by any form of delegation, even *optimal delegation*.

[4] Note that the multiplicity of solutions for the quadratic loss case if θ is public information is eliminated, because fully revealing communication now requires that $c^* = 1$.

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Online Appendix

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Preliminaries:

Let $Y = (Y_1, Y_2)$ be an n-dimensional Normal random variable with

$$\mu = (\mu_1, \mu_2)\,, \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where the dimensions of Y_1, μ_1 and Σ_{11} are m, m, and $m \times m$. The conditional distribution of $(Y_1|Y_2 = y_2)$ is Normal with conditional mean vector

$$\mathbb{E}\left[Y_1|Y_2 = y_2\right] = \mu_1 + (y_2 - \mu_2)\,\Sigma_{22}^{-1}\Sigma_{21} \tag{A1}$$

and conditional covariance matrix satisfying

$$\Sigma^* = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}. \tag{A2}$$

Applying equation $(A1)$, the conditional expectations are

$$\mathbb{E}\left[\eta|s_\omega, s_\eta\right] = \alpha^s s_\omega + \beta^s s_\eta \tag{A3}$$

and

$$\mathbb{E}\left[\omega|\, s_\omega, s_\eta\right] = \alpha^r s_\omega + \beta^r s_\eta, \tag{A4}$$

where the weights in the sender's ideal choice are

$$\alpha^s = \sigma_{\varepsilon_\eta}^2 \frac{\rho\sigma^2}{(\sigma^2 + \sigma_{\varepsilon_\omega}^2)(\sigma^2 + \sigma_{\varepsilon_\eta}^2) - (\rho\sigma^2)^2}$$

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and

$$\beta^s = \sigma^2 \frac{\sigma^2_{\varepsilon_\omega} - \sigma^2\rho^2 + \sigma^2}{(\sigma^2 + \sigma^2_{\varepsilon_\omega})(\sigma^2 + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma^2)^2},$$

and the weights in the receiver's ideal choice are

$$\alpha^r = \sigma^2 \frac{\sigma^2_{\varepsilon_\eta} + \sigma^2 - \sigma^2\rho^2}{(\sigma^2 + \sigma^2_{\varepsilon_\omega})(\sigma^2 + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma^2)^2}$$

and

$$\beta^r = \sigma^2_{\varepsilon_\omega} \frac{\sigma^2\rho}{(\sigma^2 + \sigma^2_{\varepsilon_\omega})(\sigma^2 + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma^2)^2}.$$

Proof of Lemma 1. The second moments of the random variable θ can be calculated as

$$C = \alpha^s\sigma^2 + \beta^s\rho\sigma^2 = \rho\sigma^2 \frac{\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2} + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2} + 1 - \rho^2}{\left(1 + \frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2}\right)\left(1 + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\right) - \rho^2},$$

which for $\sigma^2_{\varepsilon_\omega} = 0$ amounts to

$$C = \rho\sigma^2.$$

Similarly,

$$\begin{aligned} V &= (\alpha^s)^2 Var(s_\omega) + (\beta^s)^2 Var(s_\eta) + 2\alpha^s\beta^s Cov(s_\omega, s_\eta) \\ &= (\alpha^s)^2 \left(\sigma^2 + \sigma^2_{\varepsilon_\omega}\right) + (\beta^s)\, 2 \left(\sigma^2 + \sigma^2_{\varepsilon_\eta}\right) + 2\alpha^s\beta^s\rho\sigma^2 = \sigma^2 \frac{\frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2} + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\rho^2 + 1 - \rho^2}{\left(1 + \frac{\sigma^2_{\varepsilon_\omega}}{\sigma^2}\right)\left(1 + \frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\right) - \rho^2}, \end{aligned}$$

which for $\sigma^2_{\varepsilon_\omega} = 0$ amounts to

$$V = \sigma^2 \frac{\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2}\rho^2 + 1 - \rho^2}{\frac{\sigma^2_{\varepsilon_\eta}}{\sigma^2} + 1 - \rho^2}.$$

For $\sigma^2_{\varepsilon_\eta} \to 0$, we get $V = \sigma^2$. Applying l'Hospital, for $\sigma^2_{\varepsilon_\eta} \to \infty$ we get $V = \rho^2\sigma^2$. ■

Lemma A1 *Expected losses are minimized for the receiver and for the sender for an action* $x^* = \mathbb{E}[y|\, s_\omega, s_\eta]$ *for* $y = \omega, \eta$, *respectively.*

Proof of Lemma A1. Let $u^s(\cdot) = u^r(\cdot) \equiv u(\cdot) = -\ell(\cdot)$ and $y = \omega, \eta$. Consider the problem

$$\max_x \int_{-\infty}^{\infty} u(x-y)\, f(y|\, s_\omega, s_\eta)\, dy,$$

where $f(y|\, s_\omega, s_\eta)$ is the conditional density of $y = \omega, \eta$ given the signals. Since the utility depends only on the distance between x and y we have $u'(x-y) > 0$ for $y > x$, $u'(x-y) = 0$ for $x = y$, and $u'(x-y) < 0$ for $y < x$.

Consider the candidate solution $x^* = \mu_y \equiv \mathbb{E}[y|\, s_\omega, s_\eta]$. The first-order condition can be written as

$$\int_{-\infty}^{\infty} u'(x^* - y)\, f(y|\, s_\omega, s_\eta)\, dy = \int_{-\infty}^{\infty} u'\left(\mu_y - y\right) f(y|\, s_\omega, s_\eta)\, dy = 0.$$

Consider two points $y_1 = \mu_y - \Delta$ and $y_2 = \mu_y + \Delta$ for arbitrary $\Delta > 0$. By symmetry of u around its bliss point and symmetry of the distribution around μ_y, we have

$$u'(\Delta)\, f\left(\mu_y - \Delta\middle|\, s_\omega, s_\eta\right) = -u'(-\Delta)\, f\left(\mu_y + \Delta\middle|\, s_\omega, s_\eta\right).$$

Since this holds point-wise for each Δ, it also holds if we integrate over Δ. Thus, the first-order condition is satisfied at $x^* = \mu_y$. By concavity of u in x, only one value of x satisfies the first-order condition. ■

Proof of Proposition 1. Recall that $u(\cdot) = -\ell(\cdot)$. An optimal information structure solves:

$$\max_{V \in [\rho^2\sigma^2, \sigma^2]} \int u\left(z\left(\sigma^2 - \frac{(\rho\sigma^2)^2}{V}\right)^{\frac{1}{2}}\right)\phi(z)\, dz + \int u\left(\left(\frac{(\rho\sigma^2)^2}{V} - 2\left(\rho\sigma^2\right) + \sigma^2\right)^{\frac{1}{2}} t\right)\phi(t)\, dt.$$

The derivative wrt V is

$$\begin{aligned}
&\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{-\frac{1}{2}}u'\left(z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{\frac{1}{2}}\right)\phi\left(z\right)dz\\
&-\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{-\frac{1}{2}}tu'\left(\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{\frac{1}{2}}t\right)\phi\left(t\right)dt.
\end{aligned} \tag{A5}$$

First, suppose $V=\rho\sigma^2$. Then, the derivative wrt V satisfies

$$\begin{aligned}
&\int\frac{1}{2}z\left(\sigma^2-\rho\sigma^2\right)^{-\frac{1}{2}}u'\left(z\left(\sigma^2-\rho\sigma^2\right)^{\frac{1}{2}}\right)\phi\left(z\right)dz\\
&-\int\frac{1}{2}\left(-\rho\sigma^2+\sigma^2\right)^{-\frac{1}{2}}tu'\left(\left(-\rho\sigma^2+\sigma^2\right)^{\frac{1}{2}}t\right)\phi\left(t\right)dt\\
=\ &0.
\end{aligned}$$

Now suppose $V\neq\rho\sigma^2$. Note that both integrands in (A5) have the common representation

$$\int\frac{1}{a}ku'\left(ak\right)\phi\left(k\right)dk. \tag{A6}$$

Differentiating wrt a, we observe that (A6) is monotone decreasing in a,

$$-\frac{1}{a^3}\int aku'\left(ak\right)\phi\left(k\right)dk+\frac{1}{a^3}\int a^2k^2u''\left(ak\right)\phi\left(k\right)dk\leq 0,$$

where the inequality follows from the curvature condition

$$q\frac{u''\left(q\right)}{u'\left(q\right)}=q\frac{\ell''\left(q\right)}{\ell'\left(q\right)}\geq 1. \tag{A7}$$

$V<\rho\sigma^2$ implies $\frac{\left(\rho\sigma^2\right)^2}{V}-2\rho\sigma^2+\sigma^2>\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}$. The curvature condition (A7) implies monotonicity and therefore

$$\begin{aligned}
&\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{-\frac{1}{2}}u'\left(z\left(\sigma^2-\frac{\left(\rho\sigma^2\right)^2}{V}\right)^{\frac{1}{2}}\right)\phi\left(z\right)dz\\
\geq\ &\frac{1}{2}\frac{\left(\rho\sigma^2\right)^2}{V^2}\int\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{-\frac{1}{2}}tu'\left(\left(\frac{\left(\rho\sigma^2\right)^2}{V}-2\left(\rho\sigma^2\right)+\sigma^2\right)^{\frac{1}{2}}t\right)\phi\left(t\right)dt.
\end{aligned}$$

Hence the derivative is non-negative for $V < \rho\sigma^2$. By symmetry, the derivative is non-positive for $V > \rho\sigma^2$. These inequalities become strict for functions that satisfy the curvature condition (A7) with strict inequality. It follows that the problem is maximized in V for $V = \rho\sigma^2$. ■